

REMARKS

By the foregoing, claim 11 is amended in view of the rejection under 35 USC § 101. A comma is inserted in claim 7. Non-elected claims 12-16 are cancelled.

Favorable reconsideration of the application in its presently-amended form is requested for reasons discussed hereinbelow.

Interview Request

At the outset, a personal interview with the Examiner is requested in view of the continued rejection. The undersigned will telephone the Examiner to schedule an interview.

Restriction Requirement

The restriction requirement of August 9, 2006 has been made final. Group I claims 2-11 and 17-19 are being examined. Group II claims 12-16 are now cancelled.

Again, it is noted with appreciation that the Examiner considers the inventions of Groups I and II to be patentably distinct. Accordingly, it is assumed that a consistent standard of patentability will be applied when examining the claims in view of the prior art.

35 USC § 101

Claim 11 continues to be rejected under 35 USC § 101 on the asserted basis that the claim is directed to non-statutory subject matter. The Examiner asserts: "A deeper look into the specification reveals the production of 'a fiber length distribution' is merely a series of mathematical algorithms carried out within a processor and the analyzed result of this calculation is not output anywhere enabling said result to become useful and concrete as stated above."

In response, claim 11 is amended to recite "outputting the fiber length distribution" as a final method step.

For the record, a "distribution" is a useful measurement. Please refer to the attached Wikipedia article "Distribution (mathematics)," url [http://en.wikipedia.org/wiki/Distribution\\_%28mathematics%29](http://en.wikipedia.org/wiki/Distribution_%28mathematics%29).

Specification paragraph [0004] reads: "The terminologies complete fiber length distribution, fiber length histogram, and fiber length probability density function (PDF) are interchangeably used herein and in the literature on fiber length measurements." Specification paragraph [0005] goes on to refer to the importance of cotton fiber length measurements in determining commercial market values of cotton. Further, specification paragraph [0010] refers to "data products."

To conclude, "fiber length distribution" is an important measurement or data product which clearly is useful and of practical commercial significance. Claim 11 now recites that the result of the calculation is output. The rejection of Claim 11 under 35 USC § 101 should now be withdrawn.

Prior Art Rejections  
Office Action Sections 4 and 5

In numbered section 4 of the Office Action, Claims 2-11 are rejected under 35 USC § 103 as unpatentable over Ghorashi et al Pat. No. 5,907,394 ("Fiber Strength Testing System") in view of Shofner et al Pat. No. 5,539,515 ("Apparatus and Methods for Measurement and Classification of Trash in Fiber Samples"). In numbered section 5 of the Office Action, claims 3-5 are rejected under 35 USC § 103 as unpatentable over Ghorashi et al Pat. No. 5,907,394 in view of Shofner et al Pat. No. 5,539,515, further in view of Shofner al PCT application Publication Number WO 01/20321 ("Conditioning and Testing Cotton Fiber").

As recognized by the Examiner, Ghorashi et al "fails to show ... acquiring a two dimensional image of the tapered beard and determining fiber amount as a function of one-dimensional distance x from the fiber sampler by averaging across the tapered beard imaged." (What is relevant for present purposes, Ghorashi et al do disclose an apparatus for determining Hertel's "amount"

(A) as referred to in paragraph [0002] of the subject specification, differing little from Hertel's 1942 disclosure (Hertel Pat. No. 2,299,983). The Examiner also now refers to Ghorashi et al column 10, line 49 through column 11, line 12, for a disclosure of "a digital computer 200 connected to an output of said optical imaging device (CCD) for storing two-dimensional image data.")

The Examiner's position with reference to Shofner et al Pat. No. 5,539,515 is understood to be that '515 Shofner "discloses using a two-dimensional representation of a fiber in order to determine the diameter of a fiber versus its length." The implication of the Examiner's characterization is understood to be that "two-dimensional representation" means imaging. Correlating to the language of claim 2, for example, it is further understood that the Examiner views fiber length in '515 Shofner as corresponding to the claimed "one-dimensional distance x from the fiber sampler"; and views fiber diameter or width in '515 Shofner as corresponding to the claimed "fiber amount ... [determined] by averaging across the width of the tapered beard as imaged."

The Examiner's conclusion is understood to be: "It would have been obvious to someone of ordinary skill in the art to combine the device of Ghorashi with the two-dimensional imaging of Shofner for the purposes of providing a control system for sensing the physical properties of cotton as it progresses through a gin (Ghorashi, abstract)."

Reconsideration is requested for the following reasons, briefly stated:

(1) Shofner et al Pat. No. 5,539,515 does not disclose "using a two-dimensional representation of a fiber in order to determine the diameter of a fiber versus its length," if the Examiner's use of "two-dimensional representation" is intended to refer to "imaging" as in applicant's claim 2. Rather, '515 Shofner et al discloses e.g. extinction mode sensors (column 9, lines 5-31) and forward scatter sensors (column 9, lines 30-38),

but not image sensors. Fiber length is calculated based on the time it takes for a fiber to travel past the sensors.

Thus, quoting from '515 Shofner et al column 10, lines 41-67:

"From the description set forth above, it should be appreciated that TB appearing on line 212 represents the time required for the beginning of an entity, a fiber in this case, to pass from an optical projection of sensor 116 to an optical projection of sensor 118. Thus, TB corresponds to the speed of the leading edge of the entity. TE appearing on lines 214 represents the time required for the trailing end of an entity to pass from an optical projection of sensor 116 to sensor an optical projection of 118 and, thus, corresponds to the speed of the trailing end of the entity. TF appearing on lines 224 represents the time required for an entity to pass completely by an optical projection of extinction sensor 116. Thus, the TF corresponds to a dimension of the entity (such as the length of a fiber) and this dimension can be calculated based upon the speed of the entity. The signal appearing on line 234 represents the time integral of the light extinguished by the entity, or the area under the waveform, AE. The number appearing on line 236 represents the peak amount of light extinguished by the entity or to the peak amplitude PE. The count, TFS appearing on lines 246 represents the time required for the entity to pass by an optical projection of the scatter sensor 128 and corresponds to a dimension (such as length) of the entity as measured by the scatter sensor 128. The signal appearing on line 252 represents the time integral of light scattered by the entity as detected by sensor 128, AS, and the signal appearing on line 258 represents the peak amount of light scattered by the entity, PS."

Again, "imaging" as in applicant's independent claims 2, 9 and 11 is not involved.

(2) Even if Shofner et al Pat. No. 5,539,515 disclosed images of fibers, there is no indication that the diameter (width) of an individual fiber varies along its length, or, if so, that any particular meaning would follow from such a

measurement. The length and diameter (width) of individual fibers is not analogous to a fiber length measurements from a tapered beard. Quoting from claim 2, the subject invention is directed to "[a]pparatus for fiber length measurements from a tapered beard attached to a fiber sampler." Fiber length measurements from tapered beards is a very specific and well known measurement in the art. Quoting from specification paragraph [0002]: "Fiber length measurements from tapered beards were disclosed by Hertel in the 1940's. Hertel employed the term 'amount' (A) to refer to the amount (that is, number or linear density) of fibers in tapered beards and developed what he termed 'fibrograms,' which are plots of 'amount' as a function of distance from a needle sampler. Hertel worked out the theory for analyzing fibrograms and developed apparatus and methods for length measurements from the 1930s to the 1950s. In particular, Hertel disclosed the determination of a length-distribution curve based on optical analysis. See, as examples, Hertel U.S. Pat. Nos. 2,299,983, 2,404,708 and 3,057,019."

Again, all of that is very specific, and unrelated to the Examiner's proposal to measure and somehow employ the variation (if any) of the diameter (width) of an individual fiber along its length. The Examiner's proposal is irrelevant to cotton classing as is conventionally practiced.

(3) The references which the Examiner proposed to combine do not suggest the claimed invention. A relevant portion of the disclosure of Ghorashi et al Pat. No. 5,907,394 is column 16, line 6 through column 17, line 31. Light transmission measurements in particular are described beginning in column 6, around line 46. Ghorashi et al FIG. 25 illustrates a tapered beard 161. What is disclosed in Ghorashi et al is an apparatus for determining Hertel's "amount" (A) as referred to in paragraph [0002] of the subject specification, differing little from Hertel's 1942 disclosure (Hertel Pat. No. 2,299,983). This prior art technique may be referred to as length by optical analysis, or  $L_O$ . Prior art length by optical analysis measurements do not involve two-dimensional imaging, only simple extinction-mode

sensors. Such prior art measurements are further elaborated on in paragraphs [0009] and [0010] of the subject specification. The apparatus of Shofner et al Pat. No. 5,539,515 also employs simple extinction-mode and scatter-mode sensors (not two-dimensional imaging) to develop characteristic signals as fiber entities flow past the sensors, and computer analysis is then employed to develop useful measurements.

Quite simply, and quoting from claim 2, as an example, there is no suggestion in these references of "acquiring a two-dimensional image of the tapered beard; and ... determining fiber amount as a function of one-dimensional distance  $x$  from the fiber sampler by averaging across the width of the tapered beard as imaged." Image-based determinations of  $A(x)$ , referred to in applicant's disclosure as  $L_i$ , are entirely new. Ghorashi et al Pat. No. 5,907,394 discloses conventional light transmission measurements implementing conventional length by optical analysis ( $L_o$ ) of tapered beards. Shofner et al Pat. No. 5,539,515 is directed to analysis of individualized fiber entities transported past extinction- and scatter-mode optical sensors, and is not concerned with tapered beards. Tapered beards, optical channels and extinction mode amount  $A(x)$  determinations are well known, beginning with Hertel in the 1940s. The subject image-based determinations of  $A(x)$ , referred to in applicant's disclosure as  $L_i$ , are entirely new.

Prior Art Rejections  
Office Action Section 6

In numbered section 6 of the Office Action, Claims 9-10 and 18 are rejected under 35 USC § 103 as unpatentable over Shofner et al PCT application Publication Number WO 01/20321 ("Conditioning and Testing Cotton Fiber") in view of Shofner et al Pat. No. 5,539,515 ("Apparatus and Methods for Measurement and Classification of Trash in Fiber Samples").

Fundamentally, neither Shofner et al WO 01/20321 nor '515 Shofner et al disclose or suggest acquiring an image of a tapered beard, by scanner or otherwise. Shofner et al

WO 01/20321 discloses a color scanner for imaging a bulk cotton sample cut from a bale, not a tapered beard. As discussed in detail hereinabove, Shofner et al Pat. No. 5,539,515 is directed to analysis of individualized fiber entities transported past extinction- and scatter-mode optical sensors, and likewise is not concerned with tapered beards.

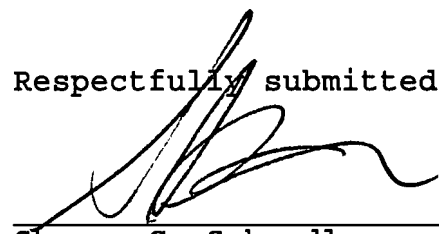
Independent claim 9 calls for "a scanner intended for scanning documents positioned with reference to the tapered beard for acquiring a two-dimensional image of the tapered beard." Quite simply, neither of these references relates to tapered beards which are well known in the art as described in detail above. There is no basis for the Examiner's stated rejection of claims 9-10 and 18, which should be withdrawn.

Conclusion

In view of the foregoing, reconsideration and allowance are requested. An interview is requested.

Group I claims 2-11 and 17-19 are in the case.

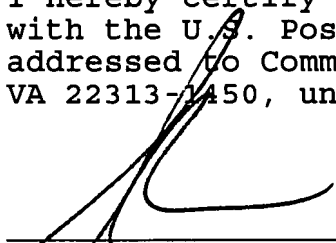
Respectfully submitted,

  
\_\_\_\_\_  
Steven C. Schnedler  
Attorney, Reg. No. 27,591

Carter Schnedler & Monteith, P.A.  
56 Central Avenue, Suite 101  
P.O. Box 2985  
Asheville, NC 28802

Telephone (828) 252-6225

I hereby certify that this paper is being deposited this date with the U.S. Postal Service as First Class Mail in an envelope addressed to Commissioner for Patents, P.O. Box 1450, Alexandria, VA 22313-1450, under the provisions of 37 CFR 1.8.

  
\_\_\_\_\_  
Steven C. Schnedler

SEPT. 5, 2007  
Date

K:\PATAMD\SEA-19 Amendment 3.doc



# Distribution (mathematics)

*You can support Wikipedia by making a tax-deductible donation.*

From Wikipedia, the free encyclopedia

*This page is about generalized functions in mathematical analysis. For the probability meaning see probability distribution. For other meanings see distribution (disambiguation).*

In mathematical analysis, **distributions** (also known as **generalized functions**) are objects which generalize functions and probability distributions. They extend the concept of derivative to all integrable functions and beyond, and are used to formulate generalized solutions of partial differential equations. They are important in physics and engineering where many non-continuous problems naturally lead to differential equations whose solutions are distributions, such as the Dirac delta distribution.

"Generalized functions" were introduced by Sergei Sobolev in 1935. They were independently introduced in late 1940s by Laurent Schwartz, who developed a comprehensive theory of distributions.

## Contents

- 1 Basic idea
- 2 Formal definition
- 3 Distributions as derivatives of continuous functions
- 4 Compact support and convolution
- 5 Tempered distributions and Fourier transform
- 6 Using holomorphic functions as test functions
- 7 Problem of multiplication
- 8 See also
- 9 References

## Basic idea

The basic idea is to identify functions with abstract linear functionals on a space of unproblematic *test functions* (conventional and well-behaved functions). Operators on distributions can be understood by moving them to the test function.

For example, if

$$f: \mathbf{R} \rightarrow \mathbf{R}$$

is a locally integrable function, and

$$\varphi: \mathbf{R} \rightarrow \mathbf{R}$$

is a smooth (that is, infinitely differentiable) function with compact support (so, identically zero outside of some bounded set), then we set

$$\langle f, \varphi \rangle = \int_{\mathbf{R}} f \varphi \, dx.$$

This is a real number which linearly and continuously depends on  $\varphi$ . One can therefore think of the function  $f$  as a continuous linear functional on the space which consists of all the "test functions"  $\varphi$ .

Similarly, if  $P$  is a probability distribution on the reals and  $\varphi$  is a test function, then

$$\langle P, \varphi \rangle = \int_{\mathbf{R}} \varphi dP$$

is a real number that continuously and linearly depends on  $\varphi$ : probability distributions can thus also be viewed as continuous linear functionals on the space of test functions. This notion of "continuous linear functional on the space of test functions" is therefore used as the definition of a distribution.

Such distributions may be multiplied with real numbers and can be added together, so they form a real vector space. In general it is not possible to define a multiplication for distributions, but distributions may be multiplied with infinitely differentiable functions.

To define the derivative of a distribution, we first consider the case of a differentiable and integrable function  $f: \mathbf{R} \rightarrow \mathbf{R}$ . If  $\varphi$  is a test function, then we have

$$\int_{\mathbf{R}} f' \varphi dx = - \int_{\mathbf{R}} f \varphi' dx$$

using integration by parts (note that  $\varphi$  is zero outside of a bounded set and that therefore no boundary values have to be taken into account). This suggests that if  $S$  is a *distribution*, we should define its derivative  $S'$  by

$$\langle S', \varphi \rangle = - \langle S, \varphi' \rangle.$$

It turns out that this is the proper definition; it extends the ordinary definition of derivative, every distribution becomes infinitely differentiable and the usual properties of derivatives hold.

**Example:** The Dirac delta (so-called Dirac delta function) is the distribution defined by

$$\langle \delta, \varphi \rangle = \varphi(0)$$

It is the derivative of the Heaviside step function: For any test function  $\varphi$ ,

$$\langle H', \varphi \rangle = - \langle H, \varphi' \rangle = - \int_{-\infty}^{\infty} H(x) \varphi'(x) dx = - \int_0^{\infty} \varphi'(x) dx = \varphi(0) = \langle \delta, \varphi \rangle,$$

so  $H' = \delta$ . Similarly, the derivative of the Dirac delta is the distribution

$$\delta'(\varphi) = -\varphi'(0).$$

This latter distribution is our first example of a distribution which is neither a function nor a probability distribution.

## Formal definition

In the sequel, real-valued distributions on an open subset  $U$  of  $\mathbf{R}^n$  will be formally defined. (With minor modifications, one can also define complex-valued distributions, and one can replace  $\mathbf{R}^n$  by any smooth manifold.) First, the space  $D(U)$  of **test functions** on  $U$  needs to be explained. A function  $\varphi: U \rightarrow \mathbf{R}$  is said to have *compact support* if there exists a compact subset  $K$  of  $U$  such that  $\varphi(x) = 0$  for all  $x$  in  $U \setminus K$ . The elements of  $D(U)$  are the infinitely often differentiable functions  $\varphi: U \rightarrow \mathbf{R}$  with compact support. This is a real vector space. We turn it into a topological vector space by stipulating that a sequence (or net)  $(\varphi_k)$  converges to 0 if and only if there exists a compact subset  $K$  of  $U$  such that all  $\varphi_k$  are identically zero outside  $K$ , and for every  $\varepsilon > 0$  and natural number  $d \geq 0$  there exists a natural number  $k_0$  such that for all  $k \geq k_0$  the absolute value of all  $d$ -th derivatives of  $\varphi_k$  is smaller than  $\varepsilon$ . With this definition,  $D(U)$  becomes a complete topological vector space (in fact, a so-called LF-space).

The dual space of the topological vector space  $D(U)$ , consisting of all continuous linear functionals  $S: D(U) \rightarrow \mathbf{R}$ , is the space of all

**distributions** on  $U$ ; it is a vector space and is denoted by  $D'(U)$ . The dual pairing between a distribution  $S$  in  $D'(U)$  and a test function  $\varphi$  in  $D(U)$  is denoted using angle brackets thus:

$$D'(U) \times D(U) \ni (S, \varphi) \mapsto \langle S, \varphi \rangle \in \mathbf{R}.$$

The function  $f: U \rightarrow \mathbf{R}$  is called **locally integrable** if it is Lebesgue integrable over every compact subset  $K$  of  $U$ . This is a large class of functions which includes all continuous functions. The topology on  $D(U)$  is defined in such a fashion that any locally integrable function  $f$  yields a continuous linear functional on  $D(U)$  whose value on the test function  $\varphi$  is given by the Lebesgue integral  $\int_U f\varphi \, dx$ . Two locally integrable functions  $f$  and  $g$  yield the same element of  $D'(U)$  if and only if they are equal almost everywhere. Similarly, every Radon measure  $\mu$  on  $U$  (which includes the probability distributions) defines an element of  $D'(U)$  whose value on the test function  $\varphi$  is  $\int \varphi \, d\mu$ .

As mentioned above, integration by parts suggests that the derivative  $\partial S / \partial x_k$  of the distribution  $S$  in the direction  $x_k$  should be defined using the formula

$$\left\langle \frac{\partial S}{\partial x_k}, \varphi \right\rangle = - \left\langle S, \frac{\partial \varphi}{\partial x_k} \right\rangle$$

for all test functions  $\varphi$ . In this way, every distribution is infinitely differentiable, and the derivative in the direction  $x_k$  is a linear operator on  $D'(U)$ . In general, if  $\alpha = (\alpha_1, \dots, \alpha_n)$  is an arbitrary multi-index and  $\partial^\alpha$  denotes the associated mixed partial derivative operator, the mixed partial derivative  $\partial^\alpha S$  of the distribution  $S \in D'(U)$  is defined by

$$\langle \partial^\alpha S, \varphi \rangle = (-1)^{|\alpha|} \langle S, \partial^\alpha \varphi \rangle \text{ for all } \varphi \in D'(U).$$

The space  $D'(U)$  is turned into a locally convex topological vector space by defining that the sequence  $(S_k)$  converges towards 0 if and only if  $S_k(\varphi) \rightarrow 0$  for all test functions  $\varphi$ ; this topology is called the weak-\* topology. This is the case if and only if  $S_k$  converges uniformly to 0 on all bounded subsets of  $D(U)$ . (A subset  $E$  of  $D(U)$  is bounded if there exists a compact subset  $K$  of  $U$  and numbers  $d_n$  such that every  $\varphi$  in  $E$  has its support in  $K$  and has its  $n$ -th derivatives bounded by  $d_n$ .) With respect to this topology, differentiation of distributions is a continuous operator; this is an important and desirable property that is not shared by most other notions of differentiation. Furthermore, the test functions (which can themselves be viewed as distributions) are dense in  $D'(U)$  with respect to this topology.

If  $\psi: U \rightarrow \mathbf{R}$  is an infinitely often differentiable function and  $S$  is a distribution on  $U$ , we define the product  $S\psi$  by  $(S\psi)(\varphi) = S(\psi\varphi)$  for all test functions  $\varphi$ . The ordinary product rule of calculus remains valid.

## Distributions as derivatives of continuous functions

The formal definition of distributions exhibits them as a subspace of a very large space, namely the algebraic dual of  $D(U)$ . It is not immediately clear from the definition how exotic a distribution might be. To answer this question, it is instructive to see distributions built up from a smaller space, namely the space of continuous functions. Roughly, any distribution is locally a (multiple) derivative of a continuous function. (The precise theorem is below.) In other words, no proper subset of the space of distributions contains all continuous functions and is closed under differentiation. This says that distributions are not particularly exotic objects; they are only as complicated as necessary.

One precise version of the theorem is the following.<sup>[1]</sup> Let  $S$  be a distribution on  $U$ . Then for every multi-index  $\alpha$ , there exists a continuous function  $g_\alpha$  such that any compact subset  $K$  of  $U$  intersects the supports of only finitely many  $g_\alpha$ , and such that

$$S = \sum_{\alpha} D^\alpha g_\alpha.$$

## Compact support and convolution

We say that a distribution  $S$  has **compact support** if there is a compact subset  $K$  of  $U$  such that for every test function  $\phi$  whose support is completely outside of  $K$ , we have  $S(\phi) = 0$ . Alternatively, one may define distributions with compact support as continuous linear functionals on the space  $C^\infty(U)$ ; the topology on  $C^\infty(U)$  is defined such that  $\phi_k$  converges to 0 if and only if all derivatives of  $\phi_k$  converge uniformly to 0 on every compact subset of  $U$ .

If both  $S$  and  $T$  are distributions on  $\mathbf{R}^n$  and one of them has compact support, then one can define a new distribution, the **convolution**  $S \square T$  of  $S$  and  $T$ , as follows: if  $\phi$  is a test function in  $D(\mathbf{R}^n)$  and  $x, y$  elements of  $\mathbf{R}^n$ , write  $\phi_x(y) = \phi(x + y)$ ,  $\psi(x) = T(\phi_x)$  and  $(S \square T)(\phi) = S(\psi)$ . This generalizes the classical notion of convolution of functions and is compatible with differentiation in the following sense:

$$d/dx (S \square T) = (d/dx S) \square T = S \square (d/dx T).$$

This definition of convolution remains valid under less restrictive assumptions about  $S$  and  $T$ . [2][3]

## Tempered distributions and Fourier transform

By using a larger space of test functions, one can define the **tempered distributions**, a subspace of  $D'(\mathbf{R}^n)$ . These distributions are useful if one studies the Fourier transform in generality: all tempered distributions have a Fourier transform, but not all distributions have one.

The space of test functions employed here, the so-called Schwartz space, is the space of all infinitely differentiable rapidly decreasing functions, where  $\phi : \mathbf{R}^n \rightarrow \mathbf{R}$  is called *rapidly decreasing* if any derivative of  $\phi$ , multiplied with any power of  $|x|$ , converges towards 0 for  $|x| \rightarrow \infty$ . These functions form a complete topological vector space with a suitably defined family of seminorms. More precisely, let

$$p_{\alpha,\beta}(\phi) = \sup_{x \in \mathbf{R}^n} |x^\alpha D^\beta \phi(x)|$$

for  $\alpha, \beta$  multi-indices of size  $n$ . Then  $\phi$  is **rapidly-decreasing** if all the values

$$p_{\alpha,\beta}(\phi) < \infty$$

The family of seminorms  $p_{\alpha,\beta}$  defines a locally convex topology on the Schwartz-space. It is metrizable and complete.

The space of **tempered distributions** is defined as the dual of the Schwartz space. In other words, a distribution  $F$  is a tempered distribution if and only if

$$\lim_{n \rightarrow \infty} \sup_{x \in \mathbf{R}^n} |x^\alpha D^\beta \phi_n(x)| = 0$$

for all multi-indices  $\alpha, \beta$  implies

$$\lim_{n \rightarrow \infty} F(\phi_n) = 0.$$

The derivative of a tempered distribution is again a tempered distribution. Tempered distributions generalize the bounded (or slow-growing) locally integrable functions; all distributions with compact support and all square-integrable functions are tempered distributions. All locally integrable functions  $f$  with at most polynomial growth, i.e. such that  $f(x) = O(|x|^r)$  for some  $r$ , are tempered distributions.

To study the Fourier transform, it is best to consider *complex*-valued test functions and complex-linear distributions. The ordinary continuous Fourier transform  $F$  yields then an automorphism of Schwartz-space, and we can define the **Fourier transform** of the tempered distribution  $S$  by  $(FS)(\phi) = S(F\phi)$  for every test function  $\phi$ .  $FS$  is thus again a tempered distribution. The Fourier transform is a continuous, linear, bijective operator from the space of tempered distributions to itself. This operation is compatible with

differentiation in the sense that

$$F(d/dx S) = ix FS$$

and also with convolution: if  $S$  is a tempered distribution and  $\psi$  is a *slowly increasing* infinitely differentiable function on  $\mathbf{R}^n$  (meaning that all derivatives of  $\psi$  grow at most as fast as polynomials), then  $S\psi$  is again a tempered distribution and

$$F(S\psi) = FS \square F\psi.$$

## Using holomorphic functions as test functions

The success of the theory led to investigation of the idea of hyperfunction, in which spaces of holomorphic functions are used as test functions. A refined theory has been developed, in particular by Mikio Sato, using sheaf theory and several complex variables. This extends the range of symbolic methods that can be made into rigorous mathematics, for example Feynman integrals.

## Problem of multiplication

The main problem of the theory of distributions (and hyperfunctions) is that it is a purely linear theory, in the sense that the product of two distributions cannot consistently be defined (in general), as has been proved by Laurent Schwartz in the 1950s.

Thus, nonlinear problems cannot be posed and thus not solved in distribution theory. In the context of quantum field theory, the non-respect of this fact is one of the sources of the "divergencies". Although in the context of the latter theory, Henri Epstein and Vladimir Glaser developed the mathematically rigorous (but extremely technical) *causal perturbation theory*, this does not solve the problem in other situations. Many other interesting theories are non linear, like for example Navier-Stokes equations of fluid dynamics.

In view of this, several theories of **algebras** of generalized functions have been developed, among which Colombeau's (simplified) algebra is maybe the most popular in use today.

## See also

- generalized function
- Colombeau algebra
- Weak solution

## References

- M. J. Lighthill (1958). *Introduction to Fourier Analysis and Generalized Functions*. Cambridge University Press. ISBN 0-521-09128-4 (defines distributions as limits of sequences of functions under integrals)
- L. Schwartz (1954), *Sur l'impossibilité de la multiplications des distributions*, C.R.Acad. Sci. Paris **239**, pp 847-848.
- <sup>1</sup> ^ Walter Rudin, *Functional Analysis* (second edition), McGraw-Hill, 1991, ISBN 0-07-054236-8.
- <sup>2</sup> ^ I.M. Gel'fand and G.E. Shilov, *Generalized Functions*, v. 1, Academic Press, 1964, pp. 103--104.
- <sup>3</sup> ^ J.J. Benedetto, *Harmonic Analysis and Applications*, CRC Press, 1997, Definition 2.5.8.

Retrieved from "http://en.wikipedia.org/wiki/Distribution\_%28mathematics%29"

Categories: Generalized functions | Functional analysis | Smooth functions

- This page was last modified 18:57, 28 August 2007.
- All text is available under the terms of the GNU Free Documentation License. (See **Copyrights** for details.)
- Wikipedia® is a registered trademark of the Wikimedia Foundation, Inc., a U.S. registered

| 501(c)(3) tax-deductible nonprofit charity.